

# Example of a modelling review

## Roof truss



Iain A MacLeod

### The Structure

Figure 1 gives an elevation and connection details for a roof truss. It is supported at each end on masonry walls.

The trusses are at 2.0m centres

### The Model

*Objectives for the model:* The model is to estimate the deflection and internal forces in the structure under permanent and non-permanent loading.

Figure 2 shows a plane frame model of the truss:

#### *Section and material properties*

Section	Area m <sup>2</sup>	I m <sup>4</sup>	E (kN/m <sup>2</sup> )	Elements	Element type
Double 100x50 timber	0.01	8.33E-6	12.0E6	1 to 22	Beam
Diagonals 6 mm diameter stainless steel cable	2.83E-5	-	297E6	23 to 36	Truss

#### *Restraints:*

Node 1 - pin

Node 2 - horizontal roller

#### *Loading*

Loading on roof: Permanent Load  $G = 1.3 \text{ kN/m}^2$   
Non-permanent load  $Q = 1.0 \text{ kN/m}^2$

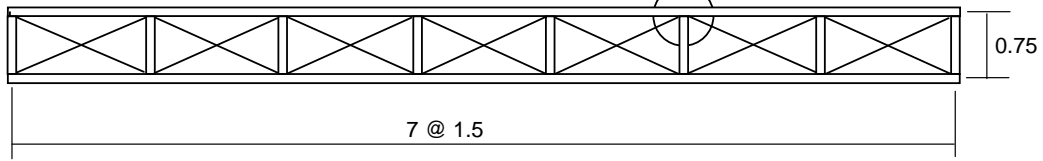
Design load for quoted case  $w = 1.35G + 1.5Q$

*Software* Strand 7

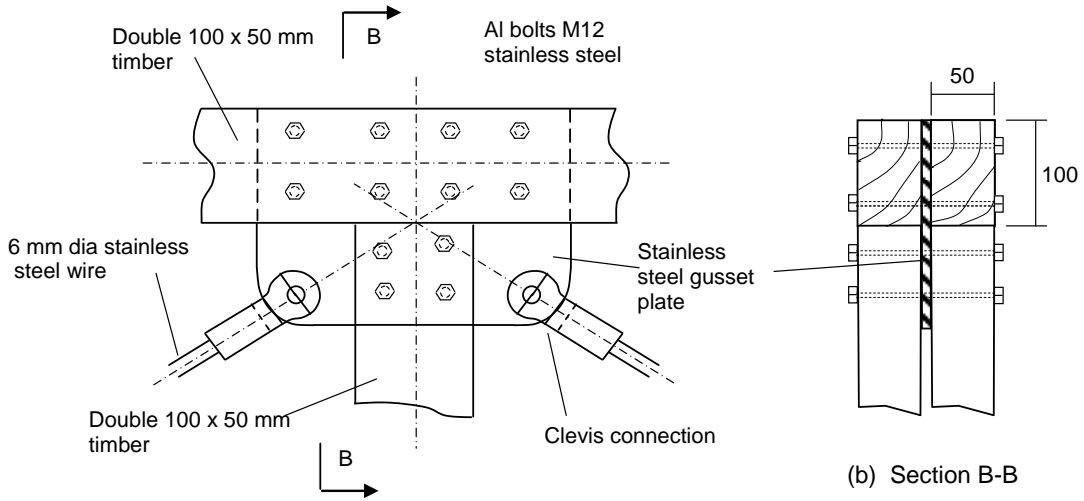
Chord members and posts are all timber double 100 x 50 mm douglas fir as shown in Detail A and Section B-B

Detail A

All diagonals are 6 mm diameter stainless steel stranded wire



(a) Elevation of truss



(b) Detail A

Figure 1 Timber truss with stainless steel bracing

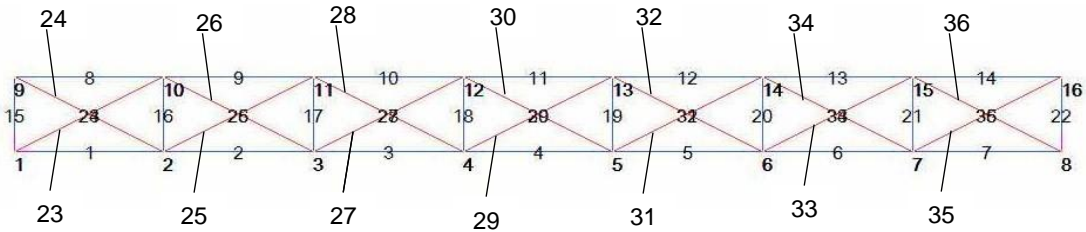


Figure 2 Model showing node and element numbers

Loading applied in model

Nodes	Applied vertical nodal loads
10 to 15	12.2 kN
9,16	6.1 kN

Node coordinates

Node	X (m)	Y (m)	Node	X (m)	Y (m)
1	0.000	0.000	9	0.000	0.750
2	1.500	0.000	10	1.500	0.750
3	3.000	0.000	11	3.000	0.750
4	4.500	0.000	12	4.500	0.750
5	6.000	0.000	13	6.000	0.750
6	7.500	0.000	14	7.500	0.750
7	9.000	0.000	15	9.000	0.750
8	10.500	0.000	16	10.500	0.750

## Results

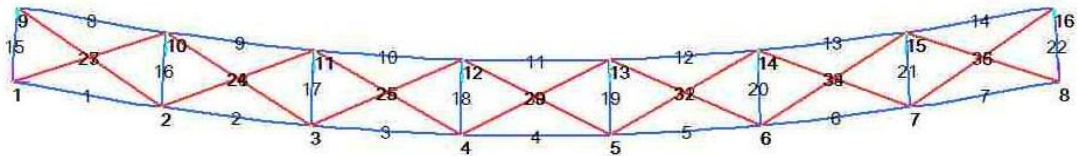


Figure 3 Deflected shape

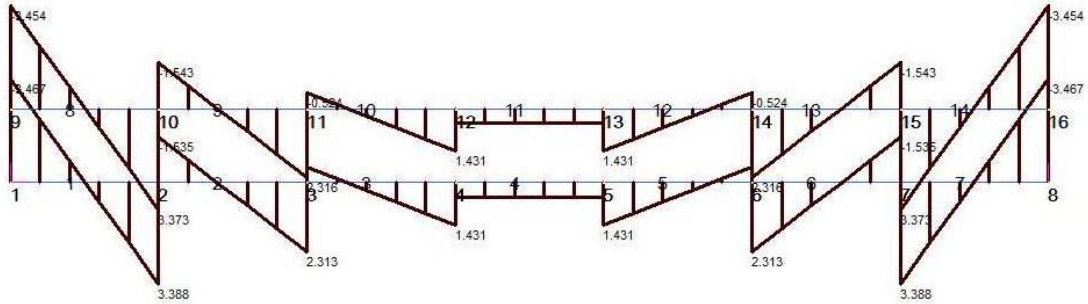


Figure 4 Bending moments in chord elements

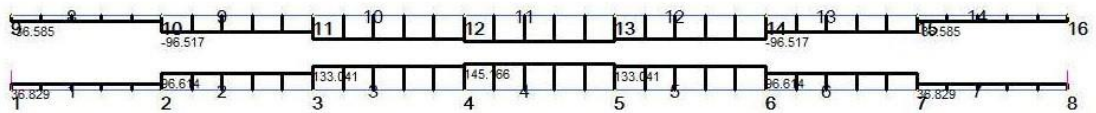


Figure 5 Axial forces in chord elements

## Sign convention for internal force actions

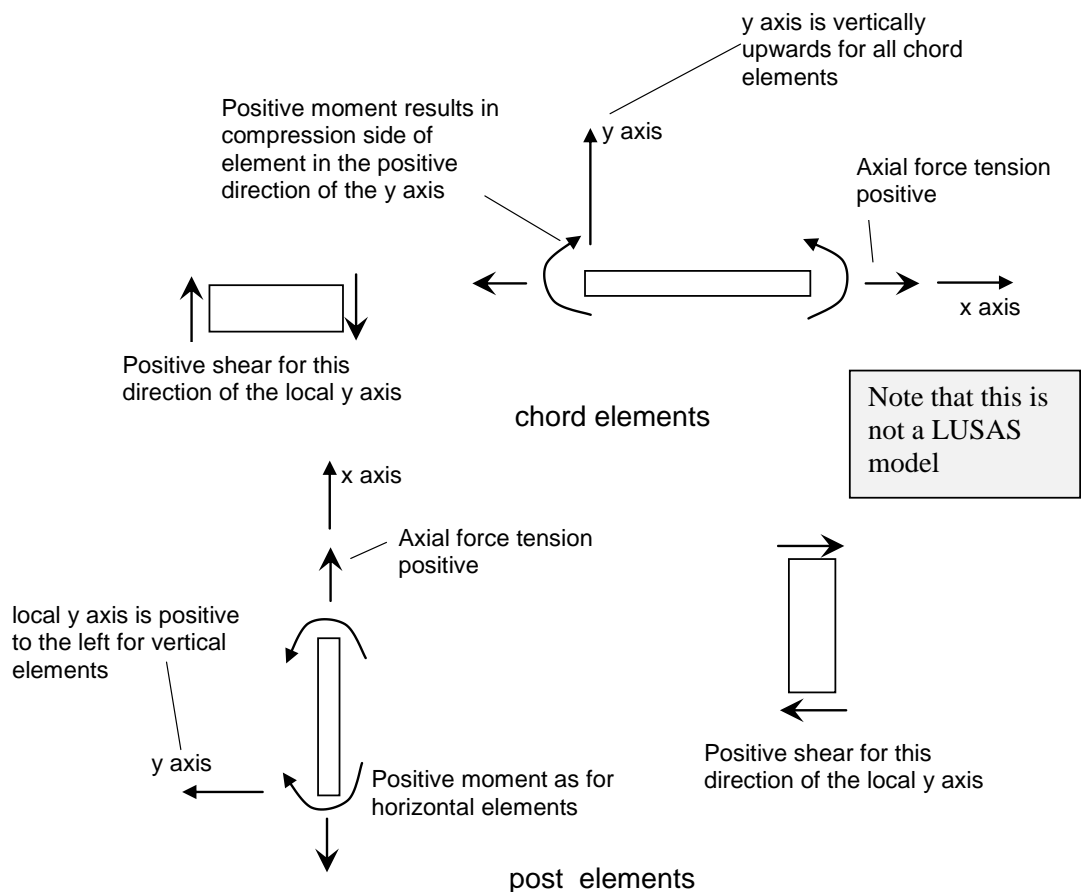


Table 1 Internal forces in elements

Beam	End	Shear	BM	Axial
Chords		(kN)	(kN m)	(kN)
1	1	4.570	-3.467	36.829
1	2	4.570	3.388	36.829
2	1	2.565	-1.535	96.614
2	2	2.565	2.313	96.614
3	1	1.305	-0.526	133.041
3	2	1.305	1.431	133.041
4	1	0.000	0.481	145.166
4	2	0.000	0.481	145.166
5	1	-1.305	1.431	133.041
5	2	-1.305	-0.526	133.041
6	1	-2.565	2.313	96.614
6	2	-2.565	-1.535	96.614
7	1	-4.570	3.388	36.829
7	2	-4.570	-3.467	36.829
8	1	4.551	-3.454	-36.585
8	2	4.551	3.373	-36.585
9	1	2.572	-1.543	-96.517
9	2	2.572	2.316	-96.517
10	1	1.303	-0.524	-132.943
10	2	1.303	1.431	-132.943

11	1	0.000	0.481	-145.069
11	2	0.000	0.481	-145.069
12	1	-1.303	1.431	-132.943
12	2	-1.303	-0.524	-132.943
13	1	-2.572	2.316	-96.517
13	2	-2.572	-1.543	-96.517
14	1	-4.551	3.373	-36.585
14	2	-4.551	-3.454	-36.585
<b>Posts</b>		<b>Shear</b>	<b>BM</b>	<b>Axial</b>
15	1	-9.2282	3.4672	-24.3294
15	2	-9.2282	-3.4540	-24.3294
16	1	-13.1182	4.9234	-6.0278
16	2	-13.1182	-4.9152	-6.0278
17	1	-7.5721	2.8388	-6.0475
17	2	-7.5721	-2.8402	-6.0475
18	1	-2.5336	0.9502	-6.0519
18	2	-2.5336	-0.9500	-6.0519
19	1	2.5336	-0.9502	-6.0519
19	2	2.5336	0.9500	-6.0519
20	1	7.5721	-2.8388	-6.0475
20	2	7.5721	2.8402	-6.0475
21	1	13.1182	-4.9234	-6.0278
21	2	13.1182	4.9152	-6.0278
22	1	9.2282	-3.4672	-24.3294
22	2	9.2282	3.4540	-24.3294
<b>Diagonals</b>				
23	1	0.000	0.000	-30.858
23	2	0.000	0.000	-30.858
24	1	0.000	0.000	30.585
24	2	0.000	0.000	30.585
25	1	0.000	0.000	-21.590
25	2	0.000	0.000	-21.590
26	1	0.000	0.000	21.482
26	2	0.000	0.000	21.482
27	1	0.000	0.000	-10.779
27	2	0.000	0.000	-10.779
28	1	0.000	0.000	10.670
28	2	0.000	0.000	10.670
29	1	0.000	0.000	-0.054
29	2	0.000	0.000	-0.054
30	1	0.000	0.000	-0.054
30	2	0.000	0.000	-0.054
31	1	0.000	0.000	10.670
31	2	0.000	0.000	10.670
32	1	0.000	0.000	-10.779
32	2	0.000	0.000	-10.779
33	1	0.000	0.000	21.482
33	2	0.000	0.000	21.482
34	1	0.000	0.000	-21.590
34	2	0.000	0.000	-21.590

35	1	0.000	0.000	30.585
35	2	0.000	0.000	30.585
36	1	0.000	0.000	-30.858
36	2	0.000	0.000	-30.858

Table 2 Displacements at nodes

Node	DX (m)	DY (m)	RZ (deg)
1	0.0000	0.0000	0.0000
2	0.0005	-0.0368	0.0005
3	0.0017	-0.0640	0.0017
4	0.0033	-0.0781	0.0033
5	0.0051	-0.0781	0.0051
6	0.0068	-0.0640	0.0068
7	0.0080	-0.0368	0.0080
8	0.0085	0.0000	0.0085
9	0.0085	-0.0002	0.0085
10	0.0080	-0.0368	0.0080
11	0.0068	-0.0640	0.0068
12	0.0051	-0.0782	0.0051
13	0.0033	-0.0782	0.0033
14	0.0017	-0.0640	0.0017
15	0.0005	-0.0368	0.0005
16	0.0000	-0.0002	0.0000

Table 3 Nodal Reactions

Node	FX (kN)	FY (kN)	MZ (kN m)
1	0.0000000000000000	42.699999999990000	0.0000000000000000
2	0.0000000000000000	0.0000000000000000	0.0000000000000000
3	0.0000000000000000	0.0000000000000000	0.0000000000000000
4	0.0000000000000000	0.0000000000000000	0.0000000000000000
5	0.0000000000000000	0.0000000000000000	0.0000000000000000
6	0.0000000000000000	0.0000000000000000	0.0000000000000000
7	0.0000000000000000	0.0000000000000000	0.0000000000000000
8	0.0000000000000000	42.699999999990000	0.0000000000000000
9	0.0000000000000000	0.0000000000000000	0.0000000000000000
10	0.0000000000000000	0.0000000000000000	0.0000000000000000
11	0.0000000000000000	0.0000000000000000	0.0000000000000000
12	0.0000000000000000	0.0000000000000000	0.0000000000000000
13	0.0000000000000000	0.0000000000000000	0.0000000000000000
14	0.0000000000000000	0.0000000000000000	0.0000000000000000
15	0.0000000000000000	0.0000000000000000	0.0000000000000000
16	0.0000000000000000	0.0000000000000000	0.0000000000000000



## Validation

**Elasticity Timber:** Acceptance criterion: design to code of practice.

*Steel wire:* Design to code of practice. Check value of E for cable (may be less than solid wire).

### Element types

*Beam elements for timber members:* Connections appear to be capable of taking moments. Timber members and connections to be designed to take combined axial forces and moments predicted by the model.

*Bending theory:* Criterion for neglecting shear deformation:  $\text{Span} : \text{depth} > 5$

Minimum span:depth =  $0.75/0.1 = 7.5$  OK

*Truss elements for the diagonals.* These elements will not take compression and therefore the compression diagonals should be removed for each loadcase. ERROR

### Connection eccentricity

Member axes do not meet at a single intersection point as shown on Detail A This cannot cause moments in the diagonals (they cannot take moment) and is unlikely to cause significant extra moments in the timber elements. OK

### Restraints

There will be some horizontal restraint at the level of the support but it is conservative to neglect it OK

### Euler buckling

Diagonals will not take compression (see above).

### Loading

Loading on roof: Permanent Load  $G = 1.3 \text{ kN/m}^2$   
Non-permanent load  $Q = 1.0 \text{ kN/m}^2$

Design load  $w = 1.35G + 1.5Q = 1.35*1.3 + 1.5*1.0 = 3.26 \text{ kN/m}^2$

Load/m on trusses =  $w*Sp = 3.26*2.5 = 8.15 \text{ kN/m}$

where Sp is the spacing of the trusses

Load at internal panel point on truss =  $8.15*1.5 = 12.2 \text{ kN}$

Load at external panel point on truss =  $12.2/2 = 6.1$

OK



## Results verification

*Data check:* Nodal coordinates - checked. Element properties - need to be checked. Loading - to be checked.

*Equilibrium of vertical load:*

Applied vertically  $6 \times 12.2 + 2 \times 6.1 = 85.4$

Sum of vertical reactions at nodes 1 and 8 -  $2 \times 42.6999 = 85.3999$  OK

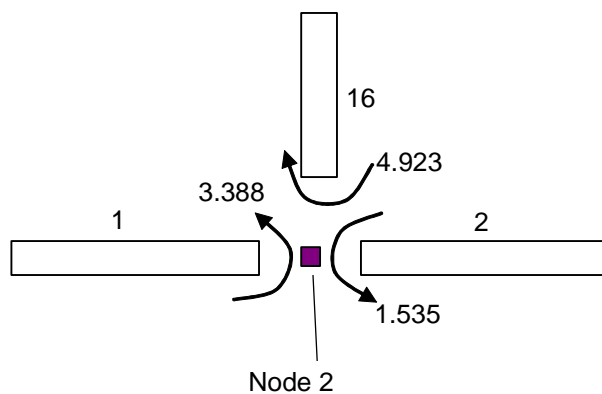
*Restraints:*

Zero deformation in X and Y directions at node 1

Zero deformation in Y direction at node 8 OK

*Symmetry:* Vertical nodal reactions = 42.6999 are the same at nodes 1 and 8 (Table 3) OK

*Check moment equilibrium at Node 2*



Sum of moments at Node 2  
(clockwise +ve)

$4.923 - 3.388 - 1.535 = 0.0$   
OK

*Form of results - displacements*

The deflected shape is curved as would be expected with UD loading. There is a significant shear deformation component because the angles between the chords and the posts deviates increasingly from a right angle from the centre to the supports.

*Form of results - internal forces*

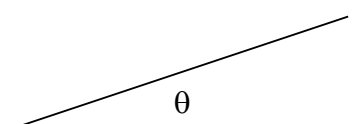
The chord bending moments increase from the centre to the supports and there are points of contraflexure in them except for the centre element where the BM is constant. Having points of contraflexure near to the centre of the beams (as in this case) is typical of vierendeel action. A (secondary) vierendeel action component is expected to occur with this type of frame.

Due to bending mode deformation the axial forces in the chord members increase from the supports to the centre as would be expected with a beam taking a UD load (BM increases towards centre of span).

The axial loads in the tie members (from Table 1) are equal and opposite in each panel and increase from the centre to the support. This is consistent with them taking shear for a UD loaded beam.

*Checking model - internal force actions*

Check forces in diagonals in panel next to supports



Total axial force in both diagonals:

$$N_d = 30.858 + 30.585$$

$$\theta = \text{atan}(0.75/1.5) = 0.464 \text{ rad}$$

$$\sin \theta = \sin(0.464) = 0.448$$

$$\text{Vert component of } N_d = N_d \sin \theta = (30.858 + 30.585) \cdot 0.448 = 27.52$$

$$\text{Shear force in this panel} = 42.7 - 6.1 = 36.5 \text{ kN}$$

Reasonable correlation since chords will take shear due to bending action.

Check axial forces in chord members at centre of span using equivalent beam model

$$\text{Moment at centre of span} = WL/8 = 83.4 \cdot 10.5/8 = 109.5 \text{ kN m}$$

$$\text{Axial force in beam } M/b = 109.5/0.75 = 146.0 \text{ kN m}$$

$$\text{Axial force in element 11} = 145.1$$

Small difference is due to axial force taken by diagonals which is small in the centre panel.

*Checking model - displacements*

Use equivalent beam model - see Information Sheet 1

$$C_b := \frac{5}{384} \quad W := 85.4 \quad L := 10.5 \quad E_t := 12.0 \cdot 10^6 \quad E_s := 197 \cdot 10^6$$

$$C_s := \frac{1}{8} \quad A_c := 0.01 \quad A_p := 0.01 \quad A_d := 2.83 \cdot 10^{-5}$$

$$b := 0.75 \quad a := 1.5 \quad L_d := (a^2 + b^2)^{.5}$$

$$\cos \theta := \frac{a}{L_d} \quad \sin \theta := \frac{b}{L_d} \quad \cotan \theta := \frac{a}{b}$$

$$f := 2.0$$

$$K_s := \frac{1}{\frac{1}{f \cdot E_s \cdot (A_d \cdot \sin^2 \theta) \cdot \cos \theta} + \frac{1}{E_t \cdot A_p \cdot \cotan \theta}} \quad I_g := A_c \cdot \frac{b^2}{2}$$

$$\Delta_b := \frac{C_b \cdot W \cdot L^3}{E_t \cdot I_g} \quad \Delta_b = 0.038 \quad \Delta_s := \frac{C_s \cdot W \cdot L}{K_s} \quad \Delta_s = 0.057$$

$$\Delta := \Delta_b + \Delta_s \quad \Delta = 0.095$$

$$\text{Computer value} = \text{nodes 4 and 5} = 0.0781$$

$$\% \text{ difference} = (0.095 - 0.0781) \cdot 100 / 0.0781 = 22\%$$

Difference due to increased stiffness of Strand model due to local bending action between chords and posts OK

**Overall assessment.** Model in general satisfactory but compression diagonals must be removed

## Checklists for Model Validation and Results Verification

### Model Validation

The model needs to be validated against the objectives of the analysis. for example overestimating deflection may not be conservative in a dynamic analysis.

- **Linear elasticity** General: for prediction of internal forces the lower bound theorem conditions should be satisfied (i.e. internal forces in equilibrium with applied load, no stress/moment greater than yield, adequate ductility). This may be satisfied by sizing of members to a code of practice.  
Steel - stress <  $f_y$ ; Concrete - for short term deformation  $f_c < f_{cu}/3$ , for long term deformation - not valid

- **Bending theory, shear deformation**

Span/depth ratio	Situation
>10	Bending theory good, shear deformation negligible
<10, >5	Shear deformation less insignificant but normally neglected
< 5	Shear deformation noticeable
<3	Shear deformation begins to dominate behaviour

- **Connection eccentricity and size**

*Connection eccentricity:* In truss structures where the dominant structural action is axial loading and where the axes of the members do not meet at a single intersection point at the joints - Figure A, the resulting eccentricity may cause significant moments in the truss members. This is likely to be more important where the eccentricity is out of the plane of plane trusses and in 3D truss systems.

*Connection size.* For moment connections, neglecting the finite size of the connection (Figure B) is normally conservative but with walls it may be best to take account of the finite width using rigid arms from the wall centreline to the beam ends.

- **Rotational flexibility of a moment connection.** May be non-negligible in steelwork connections but no simple criterion for this is available. Acceptance of full connection rigidity should be based on degree of stiffening in the details of the connection. If a connection is assumed to have rotational stiffness then it must be designed to take moment. With in-situ concrete construction, a moment connection would normally be accepted as rigid.
- **Foundation Restraints** For full fixity at a support the foundation should be massive and the connection detailed to take moment. For pin connections with a degree of rotational restraint, ensure that using a pin is conservative (likely to be acceptable for assessments of strength and deformation but may not be valid for dynamic analysis).

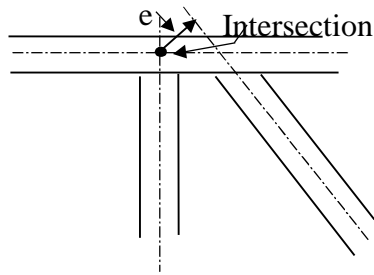


Figure A Axial force eccentricity

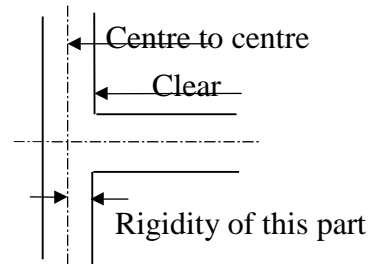


Figure B Beam to column connection

- **Small deformations including Euler buckling effect.** This assumption is normally valid due to use of code of practice rules for member sizing. For no-sway buckling of members results can be tested using the criterion:

$$\lambda = N/N_{cr} < 0.1$$

where  $N$  is the axial load and  $N_{cr,euler}$  is the Euler buckling load.

$$N_{cr,euler} = \pi^2 EI / (C_E L)^2 \quad \text{where } I \text{ is the minor axis } I \text{ value, } L \text{ is the length between connections}$$

Typical values for the factor  $C_E$  are given in Table 1.

Table 1  $C_E$  Values for Euler Buckling

End conditions	k
fixed- free (cantilever)	2.0
pin - pin	1.0
fixed - pin	.85
fixed - fixed	0.7
partial - partial	0.8 5

For overall buckling check  $\lambda = N/N_{cr} < 0.1$  where  $N$  is the total load on the system and  $N_{cr}$  is the global buckling load.

- **Loading** Acceptance criteria for loading may be based on code or practice requirements but in non-standard situations the validity of the code methods for defining the loading may need to be questioned. In non-standard situations the validity of the loading may need to be assessed by testing (e.g. wind tunnel tests).

### Results Verification

Verifying the results implies an attempt to answer the question “Has the model been correctly implemented?” The following items may be checked if relevant:

- Data check
- Sum of reactions = 0.0
- Restraints - no deformations at restrained freedoms
- Symmetry - check symmetry for symmetric structures with symmetrical loading.
- Check local equilibrium
- Form of results - internal forces
- Form of results - deformations
- Checking Model - internal forces
- Checking Model - deformations

## Equivalent beam formulae for calculating the deflection of a parallel chord truss

Beam deflection

$$\Delta = \Delta_b + \Delta_s = \frac{C_b WL^3}{E_c I_g} + \frac{C_s WL}{K_s}$$

Deflection due to bending deformation

Deflection due to shear deformation

$E$  value for material of chords

$I_g = \frac{A_c b^2}{2}$

$C_b$  and  $C_s$  from Table 1

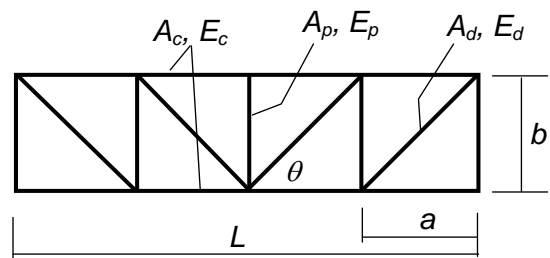
$K_s$  See Equation A or B

$W$  - total load  
 $L$  - span  
 $EI$  - stiffness parameter

$$K_{st} = \frac{1}{\frac{1}{fE_d A_d \sin^2 \theta \cos \theta} + \frac{1}{E_p A_p \cot \theta}} \quad (A)$$

If flexibility of posts is ignored:

$$K_{st} = fE_d A_d \sin^2 \theta \cos \theta \quad (B)$$



Parameters for parallel chord truss

- $f = 1.0$  for singly braced truss
- $= 2.0$  with compressive cross bracing
- $= 0.5$  for K bracing

With tensile only cross bracing, treat as singly braced.  
 With compressive cross bracing ignore flexibility of posts.

Table 1 Beam deflection coefficients

Structure	Load	$C_b$ bending	$C_s$ shear
	Point tip	1/3	1.0
	UD	1/8	1/2
	Point central	1/48	1/4
	UD	5/384	1/8

20.06.14