Case Study - Vierendeel Frame

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Contents

12.1 Vierendeel frame

12.1.1 General

This example is for a simple plane frame which demonstrates a range of modelling issues and activities. For a structure of this type a detailed modelling review as set out here may not be needed if it is commonly used. However, the results verification should be carried out as a standard procedure.
The examples should be read in conjunction with the description of the modelling process given in Chapter 3.

12.1.2 Definition of the system to be modelled - the engineering model

Portrayal

Figure 12.1(a) shows a plan of a building structure which incorporates a vierendeel roof truss arrangement fabricated using hot rolled steel sections. Figure 12.1(b) shows an elevation of a frame and Figure 12.1(c) and 12.1(d) shows details of the connections and the column baseplate support and the beam to column connection respectively.

Requirements of the model

The requirements are to estimate the displacements and internal force actions due to dead load and vertical live roof load on a frame.

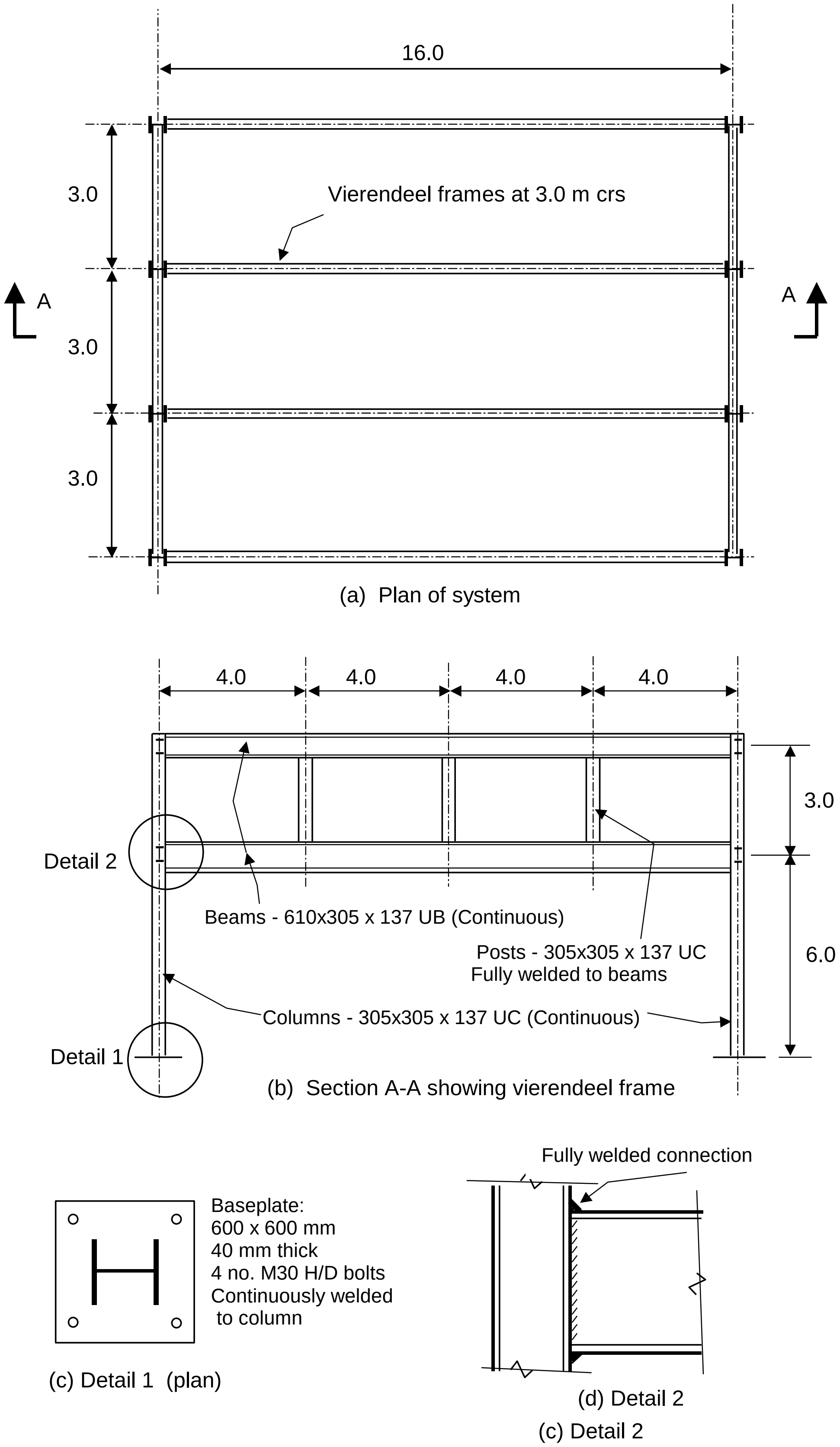

Figure 12.1 Vierendeel frame structure

12.1.3 Model development

2D or 3D model
A 3D model of the system would give better accuracy than a 2D model of a single frame but the latter is normally used and is adopted here to simplify the example for demonstration purposes.

Elements and mesh
While it would be possible to model the frame using flat shell elements the only realistic option is to use beam elements. Using engineering beam elements (Section 5.5.2) there are no discretisation issues in defining the model i.e. there is no need for mesh refinement.

Material model
The options are to use either an elastic material model or a model that allows plastic hinges to form. This context could be realistic for the latter type of model but the elastic model is chosen because availability of software.

Supports
The detail at the base of the column can take a moment but a pinned supports are used in the first instance. For the vertical loading, the sensitivity analysis (Section 12.1.7) shows that the difference between pinned and fixed supports is negligible in this case. This matter is addressed in the validation analysis (Table 12.2)

Connections
The connections are all capable of being designed as having full moment continuity so they are modelled as such. Even with the full welding there may be some local rotation due to flange distortion but full moment continuity is a conventional assumption for a frame of this type.
The finite sizes of the member could be included in the model as discussed in Section 4.5.2 but neglecting this will be conservative for displacements. Design moments for the beams should be taken at the faces of the columns and not at the column centrelines.

Loading
The loading is not specified for this context but the uncertainty in the dead loading will be low and the code values for live loading are likely to be conservative.

12.1.4 The Analysis model
Figure 12.2 shows the analysis model

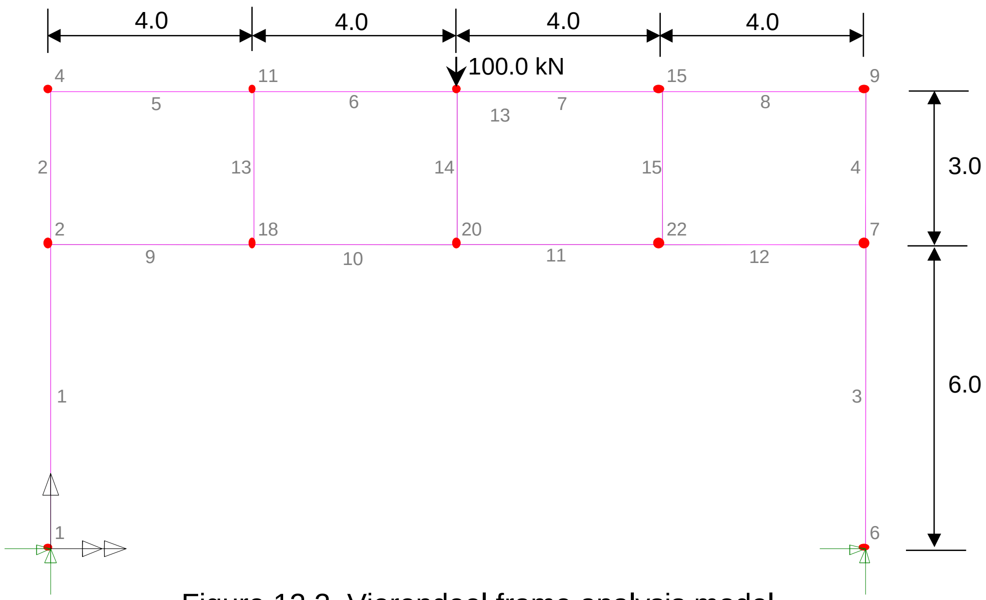

Figure 12.2 Vierendeel frame analysis model

Analysis program
LUSAS 13.3 (LUSAS 2003)

Units
Units used are metres and kilonewtons

Elements
Thick beam in-plane elements with shear deformation neglected. Properties are given in Table 12.1

Table 12.1 Element properties

Part	A_x (m^2)	A_y (m^2)	I_z (m^4)	E (kN/mm^2)
Column, posts	0.0174	4.42	0.0003281	209
Beams	0.019	7.23	0.001259	209

Support conditions
Nodes 1 and 6 are pinned.

Connections
All connections are assumed to have full moment continuity. Effect of finite widths of members at the connections (Figure 5.23) is neglected.

Non-linear geometry
Non-linear geometry effects are neglected

Loading
A nominal checking load of 100.0 kN is applied vertically downwards at node 13.

12.1.5 Model validation
The validation analysis is set out in Table 12.2

Table 12.2 Validation Analysis

Modelling issue	**Acceptance criterion**	**Outcome**
Linear elasticity	Provides a set of internal actions which can be justified for ultimate load design on the basis of the lower bound theorem (Section 2.3.3). Design to code of practice.	LSR
Bending theory, shear deformation	Criterion: L/d>10 L/d posts = 3/0.305 = 9.8 L/d beams = 4/0.610 = 6.6 On the basis of the sensitivity analysis (Section 12.1.7) include shear deformation (if software allows it)	Amend
Finite size of connections neglected	Conventional assumption; conservative for estimations of displacement. Use beam moments at faces of columns for design	CA/ LSR
Rigid moment connections	Design to code of practice	LSR
Pinned supports	Better to use a rotational stiffness of $4EI_c/h$ (BS 5950)	Amend
Non-linear geometry effect neglected	Design to code of practice	LSR

LSR - Later stage requirement; CS - criterion satisfied: CA - Conventional assumption

12.1.6 Results verification

Error warnings from software
None

Data check
No errors identified.

Sum of support reactions
Table 12.3 gives the support reactions. Accept

Table 12.3 Support reactions

Node	F_X	F_Y	M_Z
1	0.2372266714529	5.0	0.0
6	-0.2372266714529	5.0	0.0
Sum	0.0000000000000	10.0	0.0

Restraints
Table 12.4 shows that the pinned supports at nodes 1 and 6 have been properly implemented.

Table 12.4 Support deformations

Node	Δ_X	Δ_Y	θ_Z
1	0	0	2.25e-005
6	0	0	-2.25e-005

Symmetry check
The values quoted in Table 12.3 shows that the symmetry condition is satisfied (to 13 significant figures).

Qualitative check - deformations
Figure 12.3 shows the deformed mesh for the checking load. The vierendeel frame will deform in a dominant shear mode (see Section 5.10.3 and the sensitivity analysis of Section 12.1.7) which for the checking loadcase will give a straight line deflection from the centre of the span to the column. The displaced position of nodes 4, 11 and 13 are close to being in a straight line. The bending deformation of the posts does not show in Figure 12.3 but the columns bowing outwards is consistent with the rotation of the end of the end of the lower beams. No negative observations.

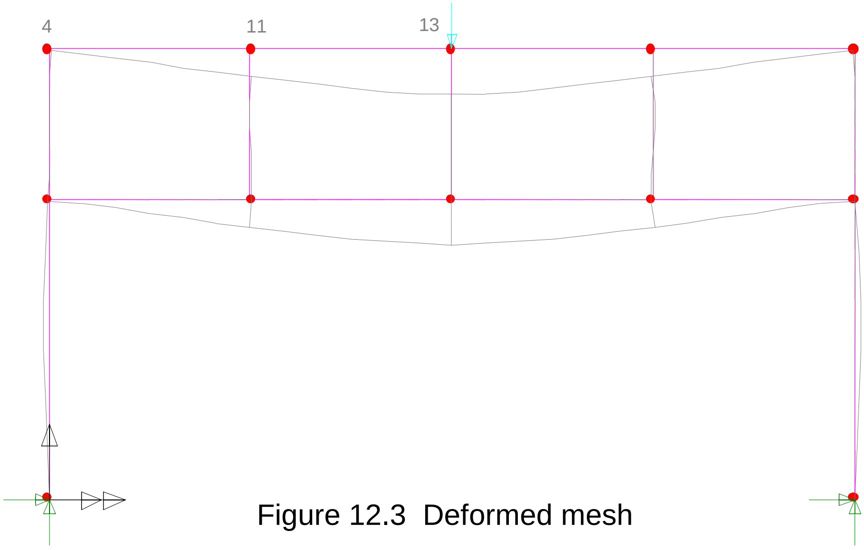

Figure 12.3 Deformed mesh

Qualitative check - internal force actions
Figure 12.4 shows the bending moments in the frame taking the checking load. Where the bending moment line crosses the longitudinal axis of the element is a point of contraflexure. Having points of contraflexure close to the mid-lengths of members is a characteristic of vierendeel frames. In the case of Figure 12.4 this tends to be so except for elements 6 and 10 (and the corresponding symmetrical pair 7 and 11). That these elements have the point of contraflexure further away form the mid positions is because:

- The posts are relatively flexible in bending as compared with the beams i.e. the ψ value is not low - see sensitivity analysis in Section 12.1.7.
- The ends of elements 6 and 10 at the centre of the span (i.e. at nodes 13 and 20) are, in effect, fully fixed i.e. there is no joint rotation for this loadcase. This localised stiffness causes moment to be drawn towards these connections pushing the points of contraflexure away from the connections.

The moments in the columns are low because of their relatively low bending stiffness (I/L value) as compared with the beam I/L.

No negative observations.

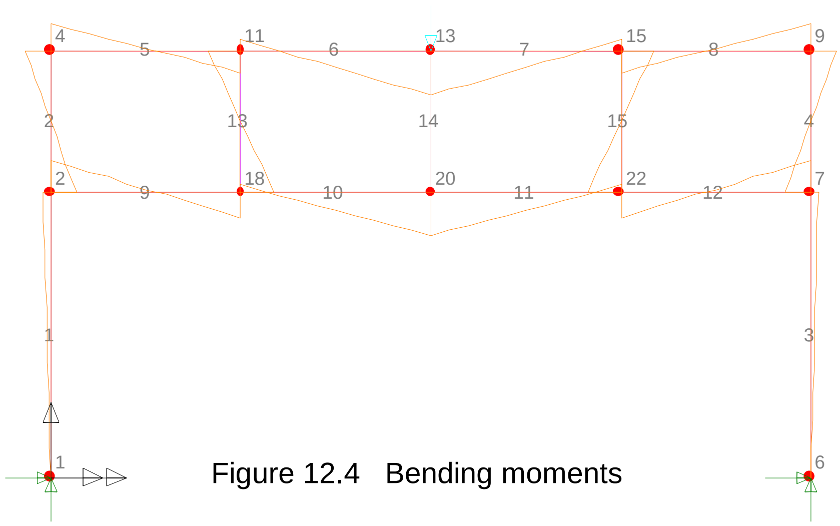

Figure 12.4 Bending moments

Checking model - deformations.
The vierendeel frame is treated as an equivalent beam - Figure 12.5 - as described in Section 5.10

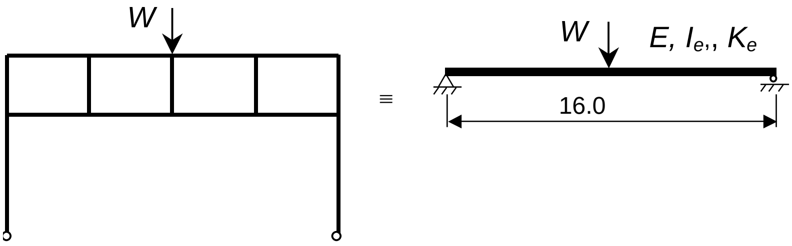

Figure 12.5 Equivalent beam model

The mid-span deflection of the equivalent beam - Δ_{frame} - using Equation (5.21):

$$\Delta_{frame} = \Delta_b + \Delta_s$$

where:

- Δ_b is the deflection due to the bending mode effect of the axial deformation of the beams.
- Δ_s is the deflection due to the shear mode effect of the bending of the beams and posts

Using the expressions for Δ_b and Δ_s from Table A4 and Equatins (5.16) and (5.24):

$$\Delta_{frame} = WL^3/(48E_cI_g) + WL/(4K_{s,vf})$$

$$I_g = A_cb^2/2 = 0.019*3\text{^}2/2 = 0.0855$$

$$\psi = (I_c/a)/(I_p/b) = (0.001259/4.0)/(0.0003281/3) = 2.88$$

$$K_s = 24E_cI_c/(a^2(1+2\psi)) = 24*209E6*0.001259/4.0\text{^}2/(1+2*2.88)$$

$$= 58387 \text{ kN}$$

hence

$$\Delta_{frame} = 100.0*16.0\text{^}3/(48*209E6*0.0855) + 100.0*16.0/(4*58387) \text{ m}$$

$$= 0.476 + 6.77 \text{ mm}$$

$$= 7.25 \text{ mm}$$

Element model value Δ_{em}

Δ_{em} = 3.97 mm

%difference $(\Delta_{eb} - \Delta_{em})/ \Delta_{em}*100$ = (3.97-7.25)/3.97*100

= -82.6%

Reasons for the difference between the two values include:

- The equivalent beam model for shear stiffness assumes points of contraflexure at the mid-lengths of all members (Section 5.11.3). This is equivalent to inserting pins into the structure to make it more flexible. The greater the real positions of the points of contraflexure deviate from the mid-length positions, the greater will be the over-estimate of deflection by the equivalent beam. It appears that the points of contraflexure not being close the centre of the mid-lengths of the beam panels in the centre of the span makes a big difference to the accuracy of the estimation of Δ_S. If the post flexural stiffness is made significantly larger than that of the beams (i.e. ψ significantly less than 1.0) then the correlation between the checking model and the element model would be much better.
- The columns provide rotational and horizontal restraints at the supports which will cause the real structure to be stiffer than the simply supported equivalent beam - see sensitivity analysis.

Correlation between the element model and the checking model is not good but the difference can be explained.

Checking model - internal force actions

The moments in the beam members at the centre of the span are estimated based on an assumption about the point of contraflexure in their lengths.

The element model results for the top and bottom beam elements at the centre of the span are given in Table 12.5

Figure 12.5 shows a free body diagram of the centre part of the truss.

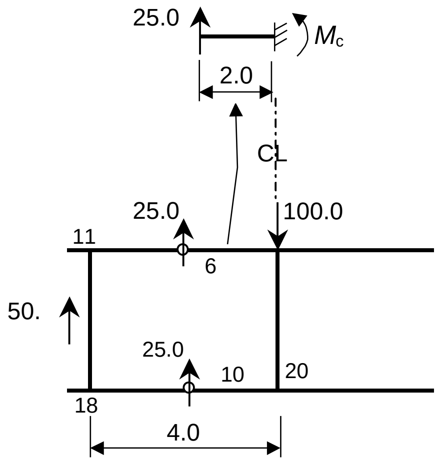

Figure 12.6 Free body diagram for shear forces at centre of the span

The shear across the frame is constant at 50.0 kN. Assuming that this is shared equally by the two beam members gives 25.0 kN in each beam. Assuming a point of contraflexure at the mid-length of the panel between the posts gives a free body diagram for the beam in the panel as shown in Figure 12.6. On this basis, the moment in the beam at the centre of the frame is:

M_c = 25.0*2.0 = 50.0 kN m

The element model value for the top beam (Element 6) from Table 12.7 is:
m_c = 85.1 kN m

Two factors that cause the moment to be underestimated by the checking model are:

- The points of contraflexure for elements 10 and 16 are not at the mid-length of the beam panel - Figure 12.5, but at a position 3.24 m from the centre of the frame in the case of the top beam.
- The shear in the top beam is greater than that in the bottom beam (due mainly to the horizontal restraining action of the columns).

Using the value of 26.2 kN for the shear in element 6 from the table gives the moment in the beam at the centre of the frame as 26.2*3.24 = 84.9 kN m. The small difference between this value and the value of 85.1 in Table 12. is due to the low number of significant figures quoted in Table 12.5

Table 12.5 End actions for elements 6 and 10

Element	**Node**	$\mathbf{n_x}$ (kN)	$\mathbf{s_y}$ (kN)	$\mathbf{m_z}$ (kN m)
6	11	73.3	26.2	19.8
6	13	-73.3	-26.2	85.1
10	18	-71.0	23.8	14.5
10	20	71.0	-23.8	80.7

Again the correlation between the checking model and the element model results is not close but can be explained. The checking model used tends to be less accurate with a small number of panels.

Review of verification outcomes
There are not negative observations in the verification. The correlation between the element model and the checking model is not good but the differences can be explained. Accept at this stage.

12.1.7 Sensitivity analysis
For the sensitivity analysis, a more realistic roof loading of 10.0 kN/m is applied vertically on the top beam .

Feature variation
A *reference model* (Section 2.4.4) - Model 1 in Table 12.6 - was used based on the system shown in Figure 12.4 with the following features:

- Pin supported columns
- No shear deformation of elements
- Loading as in Figure 12.7

The *indicative parameters* (Section 2.4.4) used are:

- Δ_{13} - the central vertical deflection of the frame at the top beam level
- S_6 - the shear in the top beam at the centre of the span
- $M_{6,13}$ - the moment in the top beam at the centre of the span

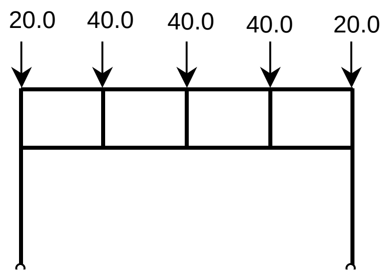

Figure 12.7 Loading for sensitivity analysis

The results from a number of feature variations are given in Table 12.6 Note that in each case only one change from the reference model is made as recommended in Section 3.4.1 The '%diff' columns in Table 12.8 give the percent difference from the reference model values i.e. %diff = (value - refvalue)/refvalue × 100

Table 12.6 Sensitivity analysis models

Model	Δ_{13} (mm)	**%diff**	s_6 (kN)	**%diff**	$m_{6,13}$ kNm	**%diff**
1. Reference	3.539		10.658		49.02	
2. No columns	3.585	1.30	10.04	-5.80	49.21	0.39
3. Shear deformation of beams	3.815	7.80	10.37	-2.70	48.67	-0.71
4. Fixed column bases	3.493	-1.30	10.74	0.77	48.93	-0.18
5. Stiff posts	2.042	-42.30	10.17	-4.58	32.36	-33.99

The models listed in Table 12.6 are

- *Model 2 No Columns* The columns were removed and simple supports were imposed at the lower level of the frame. This removes both rotational and horizontal restraint to the bottom beam members. Only small changes result.
- *Model 3 Shear deformation of the beams (only) is included* The deformation increases by 8% but the effect on the internal forces is much smaller. This is the normal effect of shear deformation in a frame. It tends to affect the deflection but tends to make little difference to the internal force actions. The result of this comparison in relation to validation is that it would be best to include shear deformation (although if the software did not allow shear deformation its exclusion could be accepted).
- *Model 4 Fixed column bases* The pin supports to the columns where changed to the fully fixed condition. This makes very little difference to the chosen indicative parameters but is likely to make a significant reduction to the moments in the columns and to the behaviour under lateral load.
- *Model 5 Stiff posts* The same section as for the beams (UB 610 x 305 x 149) was used for the posts. This makes a significant difference to the deflection and to the beam moment.

Choice of parameter variation in relation to member sizing

Model 5 of the feature variation study indicates that the span deflection is sensitive to the stiffness of the post. The question arises - "Would it be better to increase the beam stiffness or the post stiffness to stiffen the frame".

The deflection of the system is approximated by the equivalent beam relationship of Equation (5.21) and using Table A4 is:

$\Delta_{\text{frame}} = 5WL^3/(384E_cI_g) + WL/(8K_s)$

Substituting for $I_g = A_cb^2/2$ and $K_s = 24E_cI_c/(a^2(1+2\psi))$ from Equations (5.16) and (5.24) and rearranging gives:

$$\Delta_{frame} = \Delta_b + \Delta_{sb} + \Delta_{sp} = \frac{WL^3}{38.4EA_cb^2} + \frac{WLa^2}{192EI_c} + \frac{WLab}{96EI_p} \qquad (12.1)$$

where:

Δ_b is the bending mode deformation due to axial deformation of the beams
Δ_{sb} is the contribution to the shear mode deformation from the beams
Δ_{sp} is the contribution to the shear mode deformation from the posts.
W = 10.0* 16.0 = 16.0 (the total load)
Δ_b = 160*16^3/(38.4*209E6*0.019*3.0^2)*1000 = 0.478 mm
Δ_{sb} = 160*16*4.0^2/(192*209E6*0.001259)*1000 = 0.811 mm
Δ_{sp} = 160*16*4.0*3.0/(96*209E6*0.000332)*1000 = 4.612 mm
Δ_{frame} = 0.478 + 0.811 + 4.612 = 5.901 mm
Element model value Δ_{em} = 3.539 mm,
%difference = -40.0%

The difference between the equivalent beam and the element model results is less than for the point load case presumably due to the lower proportion of the total load taken as shear in the centre panels where the points of contraflexure are not close to the centre of the panels.

Taking Equation 12.1 and differentiating it successively by I_c, and I_p and substituting the reference model values (but with the UD loading) gives:

$d\Delta_{eb}/dI_c = -WLa^2/(192EI_c^2)$
= -160*16*4^2/(192*209E6*0.001259^2) = -0.644 m/m^4
$d\Delta_{eb}/dI_p = -WLab/(96EI_c^2)$
= -160*16*4.0*3.0/(96*209E6*0.00032^2) = -14.95 m/m^4

It is evident that changing the I value of the posts, rather than the I value of the beams, will be the most effective way of stiffening the frame starting from the reference configuration. One should not treat this as a general result. As the value of I_p is increased, the post will become effectively rigid in bending beyond which increases in the value of I_p will not significantly affect the frame stiffness.

12.1.8 Overall acceptance

At this point in the process there is no evidence of inadequacy in the model but the production results have yet to be generated and assessed.

12.1.9 Modelling Review Document

The information included in Section 12.1 would form a basis for a modelling review document. It is likely to be considered to have too much detail in it for a conventional design but would be needed in a safety critical or innovative situation.