Validation analysis

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Validation is assessing whether or not the model can satisfy the requirements.
The process is: List all the assumptions made for the model and consider whether they are acceptable.

Typical assumptions:

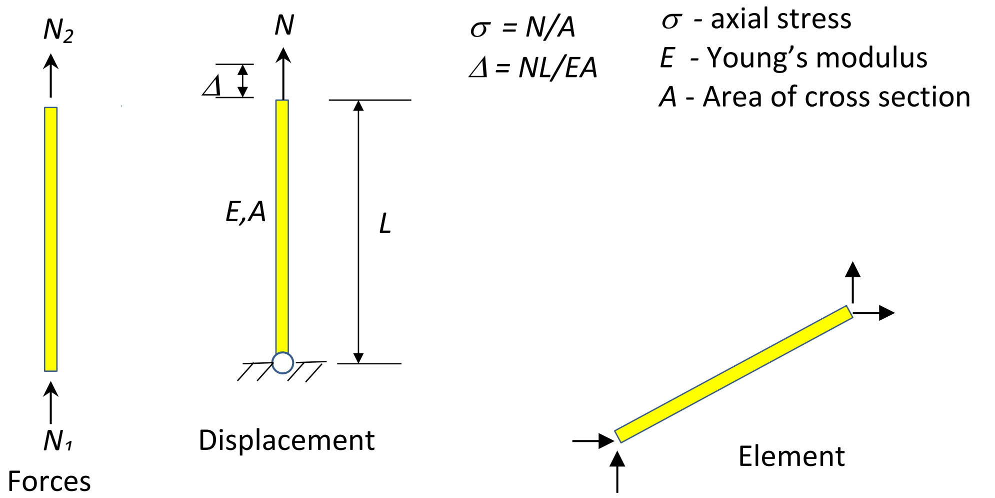
- Linear elasticity
- Small deformations - second order effects can be neglected.
- Only resultant actions
- Bar elements: no bending moment, no eccentricity of axial load.
- Bending elements: high span to depth ratios, symmetrical bending
- Loading: type, distribution, intensity
- Connections and supports : pinned, fixed, partial restraint
- Finite sizes of connections ignored.

Typical outcomes from validation questions:

- Accept
- Design to code of practice
- Conventional assumption
- Needs to be checked
- Error - Model needs to be changed

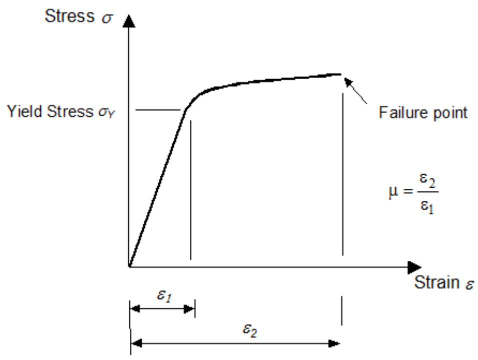
Frame elements

1. Axially loaded members - the bar element



Bar element

Tensile failure of a mild steel specimen



Assumption - Linear elasticity
Stress less than yield for ductile materials
Stress less than failure for brittle materials.

'Design to code of practice' will normally be a suitable acceptance criterion for this assumption.

Other types of material behaviour can be used e.g. plastic but solutions are non linear.

Assumption: Axially loaded elements take no bending moment. Pin connections do not transfer moment. If the flange of an I beam is not connected full moment transmission cannot be achieved.

Moment connection

(a) Photograph of a clevis pin connection

Web cleat connection
Model as a pin

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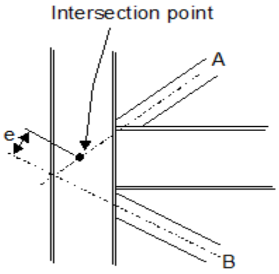
Moment connections

Web cleat connection

Assumption: Resultant of forces at the connections are in the line of the centroidal axis of the member.

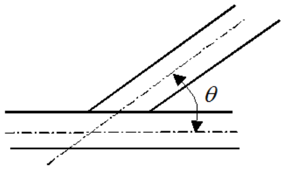


The diagonal members of the scaffolding frame (left) are out of the plane of the frame and will take a moment - as below. This is 'out of plane eccentricity' and will normally be important

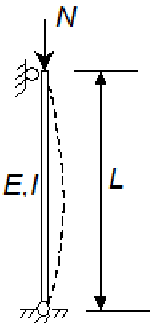
In the diagram above the axes of three of the members at the connection meet at a single intersection point. The axis of the fourth member 'B' is eccentric to this point and so there is potential for moments to be present in the connection. This is 'out of corner eccentricity'.

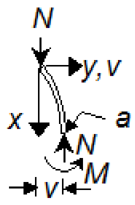
The effect of such moments can be negligible but neglect of such eccentricity was a contributing factor in the collapse of the Hartford Centre Stadium in 1971

Having a full moment connection means that the angle between the longitudinal axes of the connected members (θ) is assumed not to change under lc



Assumption: Small deformations





(b) Free body diagram of part of strut

If a strut that is not straight takes an axial load *N,* there will be a moment due the eccentricity *v* of $M = Nv$
This moment causes bending stress and lateral deflection of the strut.

If the axial load is increased, the lateral deflection also increases and if the applied moment (*Nv*) becomes equal to the value of the internal restoring moment EId^2v/dx then a state of unstable equilibrium has been reached. The strut can continue to deform in bending but does not provide further resistance to *N*. The value of *N* in this situation - N_{cr} - is the *critical load* also known as the *buckling load*.

For a pin ended strut (as shown above) the Euler critical load is given by:
$N_{cr} = \pi^2 EI/L^2$ where *L* is the length of the strut.
A more general form of this relationship is $N_{cr} = \pi^2 EI/L_E^2$
L_E is the effective length
$L_E = c_E L$ where c_E is the effective length factor - see table opposite

Effective length factors for struts

End Conditions	c_e
Restrained in position but not restrained in direction at both ends (pin-pin)	1.0
Fixed both ends (full fixed)	0.7
Fixed one end, free at other end (cantilever)	2.0
Fixed in position but partially restrained in direction at both ends	0.85

Small deformations assumption (continued)

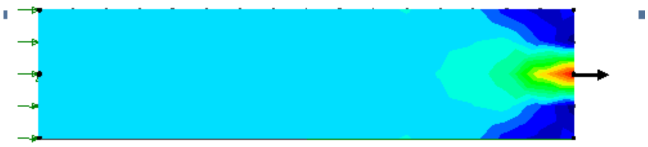
A commonly used acceptance criterion for ignoring buckling effects is that $N_{cr}/N > 10$. Using this criterion results in the increase in stress and deformation due to the eccentricity of N being not much more than 10%

The effect of eccentricity of the load in a strut is an example of *non-linear geometry* effects.

When a load is applied to a structure it changes shape. Assuming that this change can be ignored is called *first order theory* that allows linear solutions to be used. Taking account of the changes in geometry in the model is using *second order theory* for which non-linear solutions are needed. Such solutions can be achieved using modern software.

Since codes of practice take account of second order effects in the design rules for members in compression another acceptance criterion is 'Design to code of practice'.

Assumption: Only resultant actions are considered.
The diagram opposite shows a plate with a point load at the end. The contours shown are for horizontal direct stress.



(b) Contours of horizontal stress

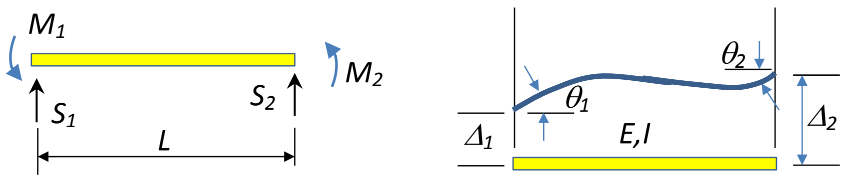
Figure 5.9 Plane stress model with point loading at the free end

The stresses are uniform over most of the plate but are not uniform close to the point load. The uniform stresses are 'resultant stresses'. Near the point load they are 'local stresses'. Axial force elements do not take account of the local stresses.

2. Members in bending - the bending element

$\sigma = MY/I$
$M = EI\, d^2v/dx^2$

σ - bending stress
E - Young's modulus
I - Second moment of area



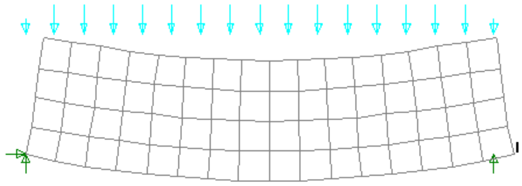
Forces

Displacements

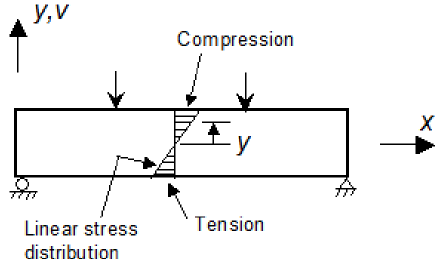
Bending element

Assumption: Linear elasticity As for axial force members

Assumption: Plane sections remain plane
The lines that were initially vertical remain straight after deformation -see right - except near the supports.



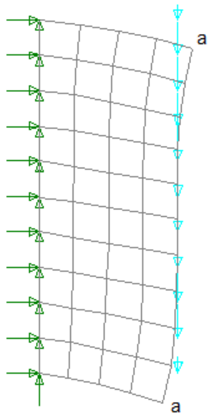
Horizontal stress is assumed to be linear (right)



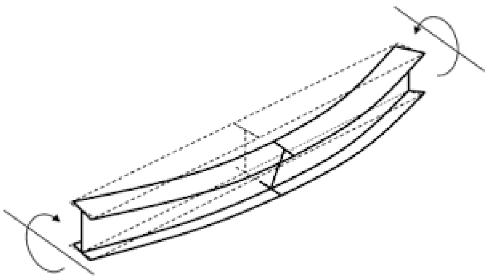
This assumption is valid for normal span to depth ratios (L/D) For low span to depth ratios ($L/D < 3$) shear deformation can be included.

Shear deformation is seldom important but can be included using 'thick beam' elements.

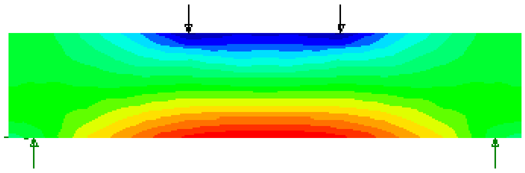
The diagram (right) shows the deformed shape of a very deep cantilever with a uniform edge load. At the support (left end) the elements deform in shear. At the loaded (right) end there is no shear deformation at the top and the bottom.



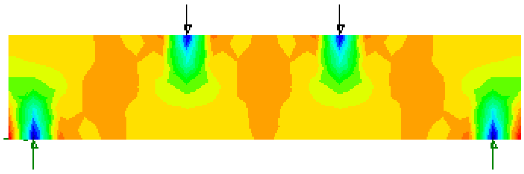
Assumption: Small deformations
No lateral torsional buckling (right)
Normal acceptance criterion - 'Design to code of practice'
Special analysis may be needed for situations that are not covered by the code.



Assumption: Only resultant actions considered
The contours shown in (b) (right) are for horizontal stress with 2 point loading on a beam. Between the loads the bending moment is constant. The horizontal stress is the due the bending moments that are resultant actions.
The contours in (c) are for vertical stress that are due to the local actions of the point loads and point reactions. Local stresses are not estimated by bending theory.

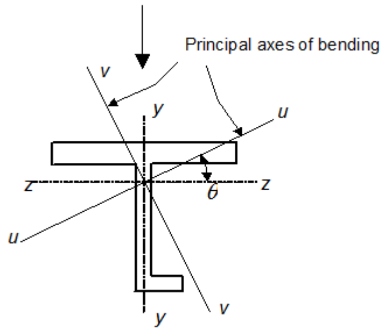


(b) Resultant Horizontal stress σ_x

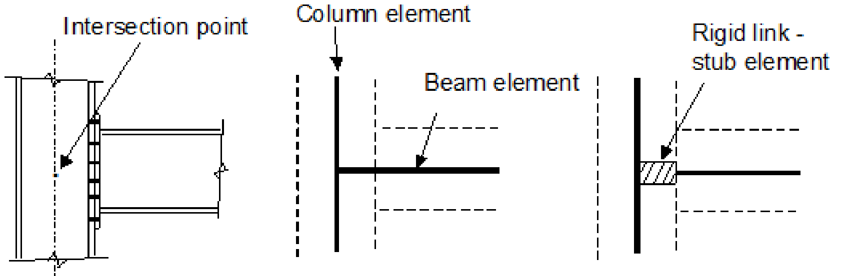


(c) Local Vertical Stress σ_y

Assumption: Symmetric bending
If the plane of loading is not a principal plane of the cross section then the orientations of the principal planes (see right) have to be identified and the loading transformed into these planes.



Assumption: Neglect finite sizes of connections
In (a) (below) is shown a beam to column connection. In (b) the model shows the members being flexible up to the intersection point. In (c) the finite width of the column is modelled as a rigid link. This is more accurate. The finite depth of the beam might also be modelled as a rigid link.

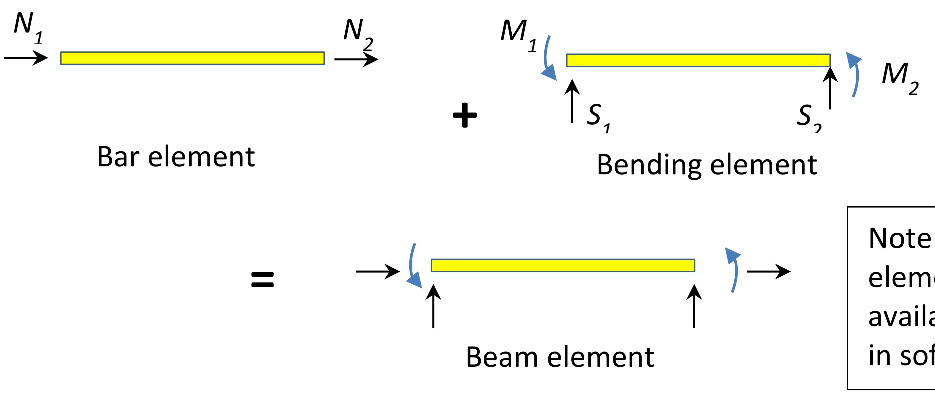


(a) Moment connection

(b) Conventional connection model

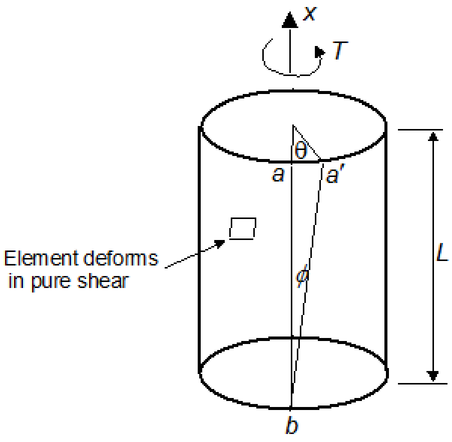
(c) Finite width of the column modelled

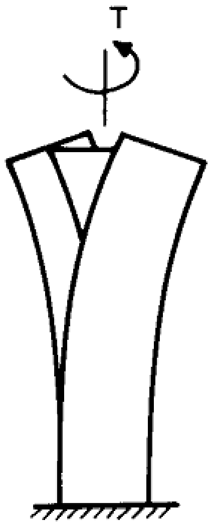
The plane beam element



Note that bending elements are not available separately in software

Torsion:
Two types: *shear torsion* and *bending torsion*





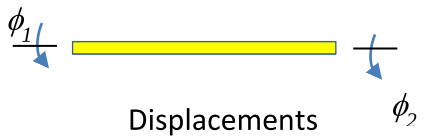
Shear torsion
For a circular bar, longitudinal straight lines remain straight after the torque is applied. The system is in pure

Bending torsion
For an I section the flanges deform in bending

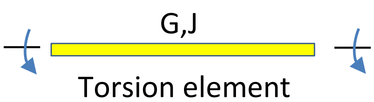
Assumption: Neglect bending torsion
For a closed section (e.g. circular or rectangular sections) shear torsion is dominant and bending torsion is normally neglected.
For an open section (such as an I or channel section) bending torsion should not be neglected if the effect of torsion is important.
A special element is needed to model bending torsion - not available in all FE software.

The torsion element

Forces



Displacements

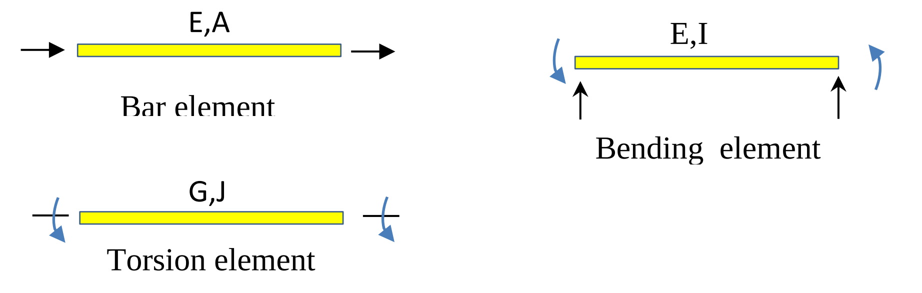


Torsion element

Note that torsion elements are not available separately in software

Types of line element

Element	Bar	Bending-Z	Bending-Y	Torsion
Bar	√			
2D Beam	√	√		
Grillage		√		√
3D Beam	√	√	√	√



Line elements

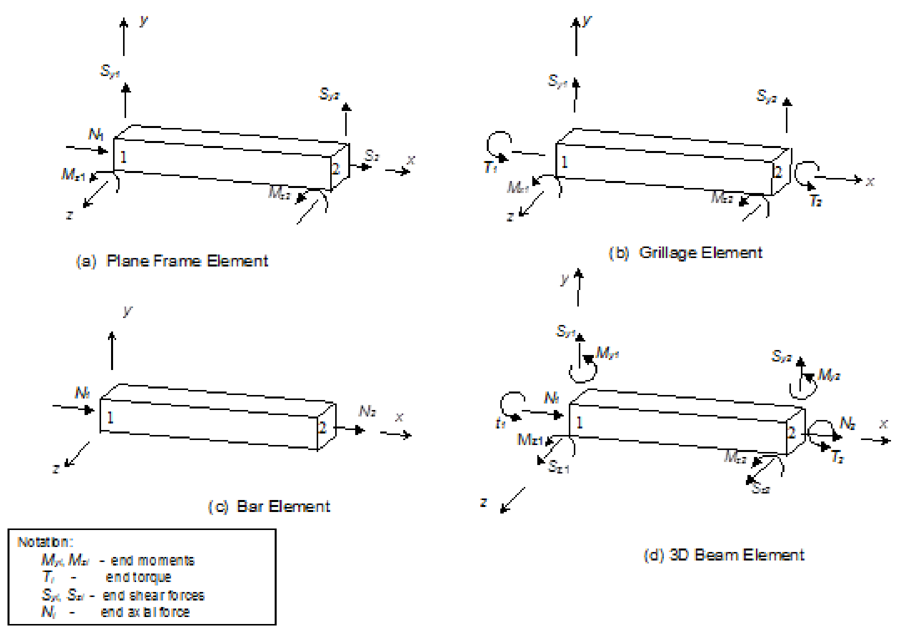


Figure 5.15 Engineering beam elements